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13. ABSTRACT (Maximum 200 words)

The main activities supported by the contract were research support (primarily travel) for C. Chekuri, B, Shepherd and P. Winkler. Funds were also used to run the Bell Labs Pow Wow workshop which addressed various problems in the field of combinatorial optimization and algorithm design. The workshop resulted in fruitful research papers and has sparked other collaborations, which may lead to new results in the future. The contract also supported a 2-week visit by Bill Cunningham who worked with G. Wilfong and B. Shepherd on some generalized coloring problems arising from the Clos-Network switch designs.
Some new technical results were enabled by the contract in the areas of All or Nothing Problems, Combinatorial Probability and Generalized Colouring and Optical Networking.

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Fundamentals of Combinatorial Optimization and Algorithm Design

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The main activities that were supported under this grant, were research support (primarily for travel) for C. Chekuri, B. Shepherd, and P. Winkler. We summarize the travel activities at the end. Second to this, funds were used to put on a small workshop over two weeks at Bell Labs, in August 2003. This workshop was unexpectedly fruitful resulting in the papers [11, 12, 2, 9] and has sparked other collaborations which may lead to new results in the future. A document [1] describing the activities of the workshop was provided as an interim report in December (can also be found at http://cm.bell-labs.com/cm/ms/who/bshep/bla.html). In addition, our group hosted a 2 week visit by Bill Cunningham who worked with G. Wilfong and B. Shepherd on some generalized coloring problems arising from Clos-Network switch designs.

We feel that the facts above have made this grant a tremendous success from our end. We would also like to discuss some of the new technical results that fall under the title of this grant, and which are enabled by our receiving this outside funding.

All or Nothing Problems: Shepherd and Vetta (IPCO 2002) considered a systematic approach to packing problems that they refer to as *demand problems*. These arise from well-known combinatorial packing problems (such as matchings) where each packed item comes with a size, and the profit is accrued only if the whole item is packed. This all-or-nothing constraint makes these problems less tractable in general. Also, with the exception of the knapsack problem and a couple of papers on unsplittable flow, there has been considerably less work on these problems, especially from the mathematical programmer's perspective.

The past ONR grants for combinatorial optimization have largely enabled Chekuri and Shepherd to follow a stream of work in this area. In last year's report we announced results [3] (due to Chekuri, Mydlarz and Shepherd) in the area of integer multicommodity demand flows in a tree (where paths/commodities come with demands). This year, based on previous results Chekuri and Shepherd found a much more general result (originally conjectured by Shepherd and Vetta) for understanding the integrality gaps for such demand problems. Suppose we have a $0,1$ matrix A and we consider the class of integer packing problems $\mathcal{P}(A, b, w)$ to be $\max\{wx : Ax \leq, x \geq 0, x \in \mathbf{Z}^n\}$. Their demand versions are: $\mathcal{P}(A, b, w, d)$ to be $\max\{wx : A[d]x \leq, x \geq 0, x \in \mathbf{Z}^n\}$, where d is an integral demand vector, and $A[d]$ is obtained form A by multiplying column i by d_i. We let $\mathcal{P}^{good}(A, b, w, d)$ be the subclass where in addition we require $d_{max} \leq b_{min}$, where $d_{max} = \max_i\{d_i\}$ and $b_{min} = \min_i\{b_i\}$. The new result proved by Chekuri and Shepherd states the following. If we know that the worst integrality gap for the relaxations for $\mathcal{P}(A, b, w)$ is at most g, then the worst gap for relaxations of $\mathcal{P}^{good}(A, b, w, d)$ is at most $12g$. This general result includes several cases previously considered. Indeed considerable energy (from several distinct teams) had been expended to show constant gaps for packing paths in the line. Our result shows this immediately (and the constants are better for

the general case) since such problems arise from totally unimodular matrices (i.e., the g involved is 1). In the final version of [3] the authors also develop a framework for tightening LP relaxations for demand problems (the setup for these is somewhat intricate, and we do not go into the details).

Also in [3], the authors define the *all-or-nothing multicommodity flow problem* for general graphs: given a capacitated graph G and demands s_i, t_i, d_i, w_i, we call a subset S of the demands *routable* if there is a multicommodity flow (not integral) in G that routes d_i units for each $i \in S$. The all-or-nothing multicommodity flow problem asks for a routable subset S that maximizes the profit $\sum_{i \in S} w_i$. One reason this problem is interesting is that it forms a relaxation for the edge disjoint paths problem EDP that has evidently not been studied previously. In EDP we are given a graph G and terminals s_i, t_i and we wish to find a maximum collection of s_i, t_i pairs that admit a collection of edge disjoint paths. In [7] it was shown that in directed graphs it is hard to approximate the optimum to within a factor of $m^{.5-\epsilon}$ for any $\epsilon > 0$, and hence EDP is terribly difficult with respect to polytime approximations. Their reduction however breaks down if small edge congestions are allowed, and also breaks down for undirected graphs. In fact, the gap for undirected graphs is only known to lie in the range 2 to $m^{.5-\epsilon}$. In [5], we show the first positive results in this direction by showing that the all-or-nothing problem does admit a poly-logarithmic (i.e., polynomial in the variable $\log(n)$) approximation for general graphs. Our techniques use recent results of Räcke on oblivious routing as well as some interesting graph theoretic clustering results in 2-connected graphs.

The above paper further inspired the authors to take a look at EDP itself, based on an insight from [5]. Using schemes of Robertson, Seymour and Thomas, they have been able to prove the following theorem in planar graphs. We call a set X *well-linked* in G, if for any subset S with $|S \cap X| \leq |X - S|$, we have $|\delta(S)| \geq |S \cap X|$. We show that if S is well-linked in a planar graph G, there is a constant C (about $10,000$ at present unfortunately) such that for any matching M of size $|S|/C$ on S, we can find paths connecting the endpoints of M, such that each edge lies in at most 3 of them. In particular, we can route any matching on S with $O(1)$ congestion in G. This result implies (again using Räcke's results) that EDP can be approximated to within a polylogarithmic factor where as n^ϵ was previously best known. Our result also shows that there is a constant approximation for product multicommodity flow in planar graphs, thus generalizing an earlier result of Klein, Plotkin and Rao for uniform multicommodity flow in planar graphs.

Combinatorial Probability

In [10], Winkler and Luczak prove the existence of, and describe, a (random) process which builds subtrees of a rooted d-branching tree one node at a time, in such a way that the subtree created at stage n is precisely a uniformly random subtree of size n. The union of these subtrees is a "uniformly random" infinite subtree, which we describe and generate in several ways. Generalization to generation of other combinatorial structures is also considered.

Generalized Colouring and Optical Networking

In [8], Winkler, Wilfong (also of Bell Labs) and Haxell (of University of Waterloo) describe a new class of graph coloring problems which arose at Bell Labs in connection with an optical networking scheme called "TWIN." In these problems the edges of a bipartite (multi)graph come equipped with *delays* which rotate colors; the basic objective is to color edges so that at nodes of one part all colors are distinct, but at nodes of the other part all colors are distinct after they have been rotated. These rotations are meant to model transmission delays for the edges involved. Even the case where the graph contains only two nodes, and parallel edges gives rise to some nontrivial mathematics (it coincides with a classical theorem of Hall).

Partial results are presented, including mention of some which have been obtained recently by Vera Asodi and Noga Alon. However, much of the paper will consist of conjectures, one of which would provide a common generalization of theorems of Denes König and Marshall Hall.

Summary of our Workshop/Conference Travel

Chekuri:

1. *Symposium on the Theory of Computation, (STOC) as part of FCRC,* June 9-11, 2003 San Diego.

2. *Thirtieth International Colloquium on Automata, Languages and Programming (ICALP)* Eindhoven, The Netherlands, June 30-July 4, 2003.

Shepherd:

1. *Network Management and Design,* IMA - Institute for Mathematics and its Applications, Minnesota, April 7-11, 2003 (most expenses covered by IMA).

2. *Constraint programming, belief revision, and combinatorial optimization* Banff International Research Station, Banff Canada, May 24-29, 2003.

3. *International Symposium on Mathematical Programming* Copenhagen, Denmark, August 18 - 22, 2003; and a preceding visit to London School of Economics, August 11-18, 2003.

Winkler:

1. *Focused Research Group: Problems in Discrete Probability,* from 07/12 to 07/26, 2003, Banff, Canada.

2. *Workshop: Markov Chain Mixing* July 25-Aug. 4 Durham, UK

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